MATHEMATICS FOR LIFE: Are you teaching students what they really need?

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Why Teach Mathematics in College?

STEM (Science, Technology, Engineering, Mathematics) students: skills of algebra, calculus, and beyond are necessary for careers

Everyone else:
 Good question!

Traditional approach: everyone takes algebra or calculus in college

Would we like everyone to be proficient with algebra and calculus? Sure.

Is it a realistic goal given core curriculum requirements? Not in 1 term.

Traditional approach: everyone takes algebra or calculus in college

And is it REALLY something these students need?

• Reasonable argument for basic algebraic skills (e.g., multiply both sides of equation)

• But most students will never again solve anything as complex as a quadratic equation.

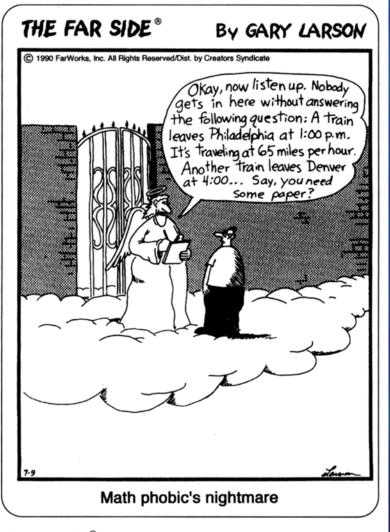
Therefore, we need a new approach...

Criteria:

 Must teach skills/concepts that students really need.

 Must be non-remedial; skills not taught on traditional pre-college algebra track.





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Litmus test — Crucial Skills must be useful for:

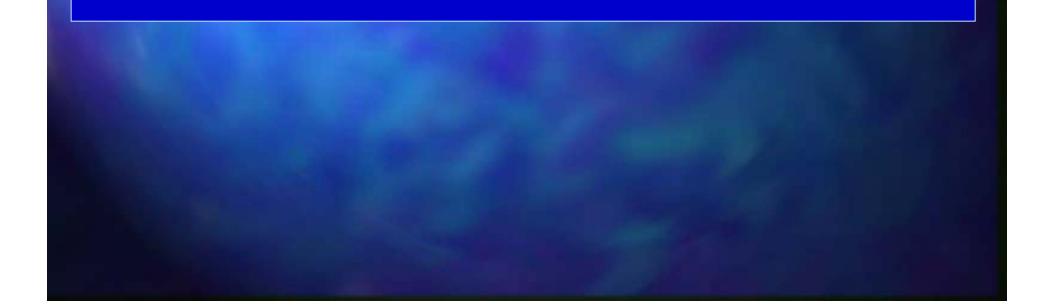
- College, especially core courses in natural and social sciences
- Careers, especially given likelihood of careers outside major area and several lifetime career changes
- Life: "citizenship issues," personal decision making and finance, etc.

So what are the crucial skills?

Logic, Critical Thinking, Problem Solving
Number Sense and Estimation
Statistical Interpretation and Basic Probability
Interpreting Graphs and Models

Together, these skills are often known as:

Quantitative Reasoning



Logic, Critical Thinking, and Problem Solving

Example: Which Airline Ticket to Buy?

You have two choices for an airline ticket:

- (A) \$1100, but 25% nonrefundable
- (B) \$1900, fully refundable

Solution notes:

- It's clearly mathematical.
- There is no "correct" answer.
- The key is to use logic and critical thinking.

Number Sense and Estimation Example: uses/abuses of percentages

From front-page New York Times: "... ...[the percentage among] 8th graders is up 44 percent, to 10.4 percent.

% of what?

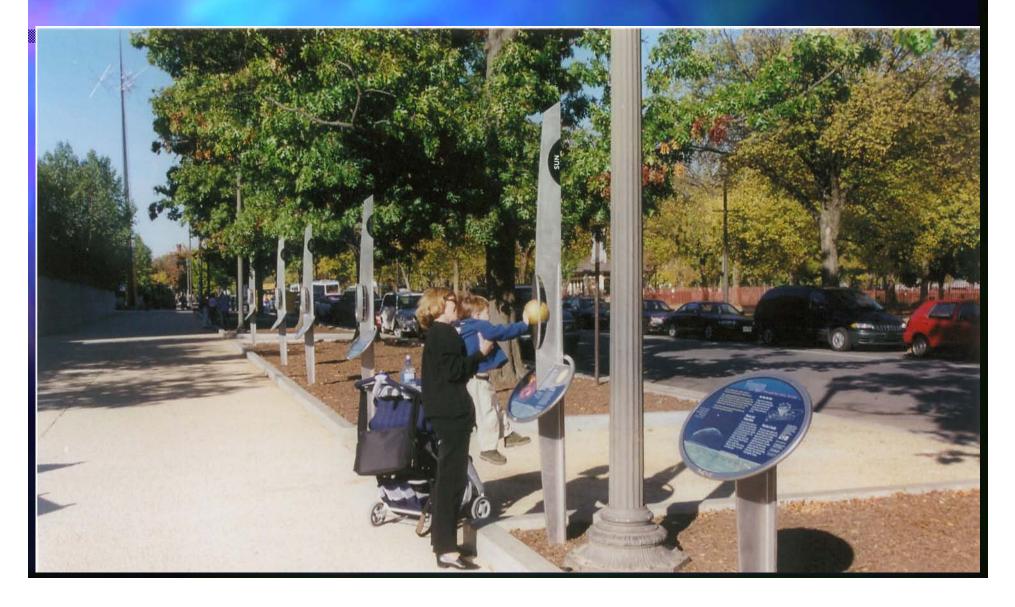
- 6/10: # jobs down 261,000

– unemployment rate *down* from 9.6% to 9.5%

Number Sense and Estimation "A billion here, a billion there, soon you're talking real money" (Everett Dirksen)

- Pro athlete earns \$1M/yr (10⁶). How long to earn \$1B (10⁹)?
- How many employees could you hire with \$100M?
- U.S. Debt \$15 trillion. What's your new baby's share?

Number Sense and EstimationExamples: How far the stars?

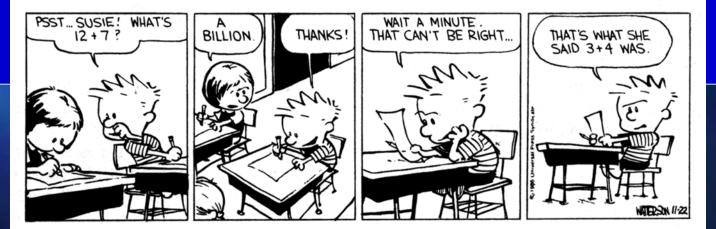


Number Sense and Estimation Dealing with uncertainty

What's the population of ...?
Drill for more oil or conserve?
What could we do with nuclear fusion power?

Calvin and Hobbes

by **Bill Watterson**



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Statistical Interpretation and Basic Probability



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Statistical Interpretation and Basic Probability

- Who's leading in the polls?
- Does correlation imply causality?
- If you're gambling losses are mounting, are you due for a win?
- Is it safe to use a hands-free cell phone while driving?

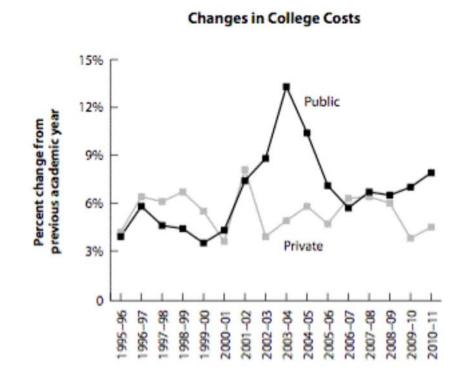
Statistical Interpretation and Basic Probability

Coincidences and pseudoscience... Example: There are 25 students in your class. What is the chance that someone else has YOUR birthday? What is the chance that two people have the same birthday?



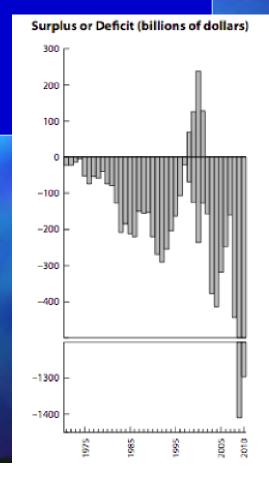
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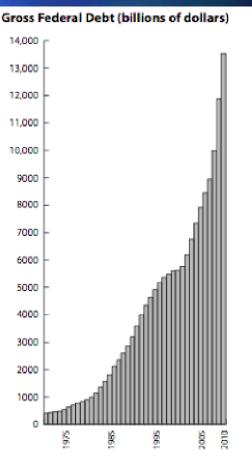
How is the cost of college changing?



Interpreting Graphs and Models

- Linear and exponential models
- Climate
- Economic models





(developed by Physics Professor Al Bartlett, U. Colorado)

Set-up

11:00: You place a single bacterium in a nutrient-filled bottle. It grows/divides into two bacteria at 11:01, 4 bacteria at 11:02, 8 bacteria at 11:03, and so on. (Doubling time = 1 minute)

12:00: Bottle is full, nutrients gone, all the bacteria die.

(developed by Physics Professor Al Bartlett, U. Colorado)

Question 1 When was the bottle *half*-full?

(developed by Physics Professor Al Bartlett, U. Colorado)

Question 2

You are a mathematically-sophisticated bacterium, and at 11:56 you recognize the impending disaster. You immediately jump on your soapbox and warn that unless your fellow bacteria slow their growth dramatically, the end is just four minutes away. Will anyone believe you?

(developed by Physics Professor Al Bartlett, U. Colorado)

Question 3

Just before disaster strikes... bacterial space program discovers three more bottles in the lab! With a population redistribution program, how much time do the 3 new bottles buy the colony?

(developed by Physics Professor Al Bartlett, U. Colorado)

Question 4

Is there any hope that future discoveries will allow the colony to continue its exponential growth indefinitely?

The Key to Success:

A context-driven rather than content-driven approach

Example – Logic. A *content*–driven approach:

We're teaching logic because it's an important area of mathematics that also happens to be useful.

- First establish mathematical ideas (e.g., sets, truth tables, Venn diagrams)
- Then discuss applications...

⇒ Problem: No obvious connection to college/career/life

The Key to Success:

Context-driven:

We're teaching logic because it's important to **college, careers, and life** — and happens to be a topic that can be addressed with mathematics.

- First establish a context; e.g., common fallacies, personal choice (airline ticket, medical insurance, cell phone plan).
- *Then* show how mathematical ideas can help.

Both approaches cover the same basic ground, but the contextdriven approach will reach a far larger percentage of the students.

Example – Loan Payments

Content-driven:

- First establish mathematical formalism of the exponential function.
- Then derive the loan payment formula and show how you can use it to calculate monthly payments.

Example – Loan Payments

Context-driven:

- *Start* with the context of personal loans and mortgages; use the loan payment formula (with or without derivation).
- When students point out that the bank does these calculations for them, build the "everyday life" part of this mathematics: decisions on closing costs, rate and term, penalties, etc.
- Later, when you are ready to teach general exponential function, point out that they've already seen these ideas.

In this case, the context-driven approach not only covers the same ground, but goes into far more depth about the real-world situations that students will face.

